OPTIMIZATION OF THERMOELASTIC LAMINAR BODIES

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Problems of laminar body synthesis are one of the promising directions in the area of structural optimization. A number of papers [1-7] concerning questions of designing laminar heat-shield panels, multilayered wave filters, elastic laminar bodies, is devoted to them. The structure of the construction and its geometric dimensions are selected as control parameters in problems of laminar construction synthesis. The control characterizing the structure of laminar bodies is a piecewise-constant function with a discrete range of values. Consequently, methods of optimal control theory, the maximum principle, finite control variations in sets of small Lebesgue measure, must be used in deriving the control equations and the construction of numerical algorithms to solve the synthesis problems. The structure and dimensions of a laminar construction are determined completely during optimization although the quantity of layers in the construction, their layer dimensions and materials are unknown in advance.

The problem of synthesis of a finite set of elastic homogeneous isotropic materials of a multilayered spherical minimum-weight shell in a stationary temperature field and loaded by internal and external hydrostatic pressure for given constraints on the strength of the sphere, its dimensions, and the critical buckling load is examined in this paper. Necessary conditions are obtained for optimality, a computational algorithm is constructed, and an example is presented of the computation of an optimal spherical shell.

1. FORMULATION OF THE PROBLEM

Let there be a set W consisting of m homogeneous isotropic materials. A laminar spherical minimum-weight shell is to be synthesized from it.

Let r_1 and r_2 be the inner and outer surface radii of the shell under consideration. For definiteness, we consider the temperature T_1 and the pressure p_1 known on the boundary r_1 while we give the heat transfer according to a Newton law and the pressure p_2 on the outer boundary r_2. The stress–strain state of a multilayered sphere under the assumption that the case of spherical symmetry holds is described by the boundary-value problem including the equilibrium equation

$$d\sigma_r/dr + 2(\sigma_r - \sigma_\varphi)/r = 0; \tag{1.1}$$

the stationary heat-conduction equation

$$\frac{d}{dr}\left(r^2\lambda\frac{dT}{dr}\right) = 0; \tag{1.2}$$

the thermoelasticity relationships

$$\begin{aligned}\sigma_r &= \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{du}{dr} + 2\nu\frac{u}{r} - \alpha(1+\nu)T\right],\\ \sigma_\varphi &= \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{u}{r} + \nu\frac{du}{dr} - \alpha(1+\nu)T\right];\end{aligned} \tag{1.3}$$

and the boundary conditions

$$\sigma_r(r_1) = -p_1,\ \sigma_r(r_2) = -p_2; \tag{1.4}$$

$$T(r_1) = T_1,\quad \lambda(r_2)\frac{dT(r_2)}{dr} = k[T_2 - T(r_2)], \tag{1.5}$$

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where $u(r)$, $\sigma_r(r)$, $\sigma_\varphi(r)$, and $T(r)$ are, respectively, the radial displacement of body points, the radial and circumferential stress components, and the stationary temperature field acting in the construction; $E(r)$, $\nu(r)$, $\alpha(r)$, and $\lambda(r)$ are the distributed characteristics of the medium, the Young's modulus, the Poisson ratio, the thermal expansion and heat conduction of the layer materials; T_2 is the temperature of the external medium; and k is the heat-transfer coefficient.

On the internal boundaries $r_i \in (r_1, r_2)$ of the layers, where the properties of the medium undergo a discontinuity, the connection conditions must be given: the continuity of the displacement $u(r)$, the radial stress $\sigma_r(r)$, the temperature $T(r)$, and the heat flux $\lambda(r)dT/dr$, i.e.,

$$[u(r_i)] = [\sigma_r(r_i)] = [T(r_i)] = \left[\lambda(r_i)\frac{dT(r_i)}{dr}\right] = 0. \tag{1.6}$$

Let σ, L, ρ_*, t_*, and λ_* be the characteristic quantities with dimensionality of the stress, length, density, temperature, and heat-conduction coefficient. Let us introduce new dimensionless variables (the asterisk on the dimensionless quantities is later omitted):

$$\begin{gathered} u^* = u/L, \quad r^* = r/L, \quad r_i^* = r_i/L, \quad p_i^* = p_i/\sigma, \quad i = 1, 2, \\ \sigma_r^* = \sigma_r/\sigma, \quad \sigma_s^* = \sigma_s/\sigma, \quad E^* = E/\sigma, \quad T^* = T/t_*, \\ \alpha^* = \alpha t_*, \quad \lambda^* = \lambda/\lambda_*, \quad \rho^* = \rho/\rho_*, \quad k^* = kL/\lambda_* \end{gathered} \tag{1.7}$$

(σ_s and ρ are the strength and density limits of the materials from the set W). Let us make the change of coordinates

$$r = r_1 + x(r_2 - r_1), \quad x \in [0, 1], \tag{1.8}$$

transferring the variable domain of assignment $[r_1, r_2]$ into the constant [0, 1]. Let us introduce the piecewise-constant function

$$\theta(x) = \{\theta_j;\ x \in [x_j, x_{j+1}),\ j = 1, \ldots, n\},\ x_1 = 0,\ x_{n+1} = 1, \tag{1.9}$$

characterizing the structure of the multilayered construction: the quantity, dimensions, and materials of the layers comprising it. The values of θ_j belong to a discrete finite set

$$U = \{\theta_1, \ldots, \theta_m\}, \tag{1.10}$$

corresponding to the given set of materials W. Now, all the characteristics of the materials from the set W will be distribution functions of $\theta(x)$ in the segment [0, 1]. It is convenient to give the set of integers $U = \{1, \ldots, m\}$ as the set U. Then writing $\theta(x) = m$, $x \in [x_k, x_{k+1})$ means that the k-th layer of the sphere consists of the m-th material of the set W.

Since the structure of the laminar spherical shell is determined by the function $\theta(x)$ and the geometry by its dimensions r_1 and r_2, we consider the pair $\{\theta(x), r_1\}$ as the control (for definiteness we consider the external radius r_2 fixed), where $\theta(x) \in U$ (1.10) and

$$r_1 \in [a, b] \tag{1.11}$$

(a, b are given limits within which the thickness of the construction under consideration can vary).

The optimal design problem is the following. Among the piecewise-constant functions $\theta(x)$ (1.9), whose range of values belongs to the set U (1.10) and the parameters r_1 from the segment $[a, b]$ (1.11), find the control $\{\theta(x), r_1\}$, achieving the minimum of the weight functional

$$F[\theta, r_1] = \int_{r_1}^{r_2} \rho(\theta) r^2 dr = \int_0^1 \Phi(x, \theta, r_1)\, dx \tag{1.12}$$

for given constraints on the strength

$$\varphi(x,\ u,\ \sigma_r,\ T,\ \theta,\ r_1)\leqslant 0 \tag{1.13}$$

and the critical buckling load

$$p_2-p_1-q(\theta,\ r_1)\leqslant 0. \tag{1.14}$$

We consider the Mises plasticity condition as the constraint (1.13)

$$\varphi(x,u,\sigma_r,T,\theta,r_1)=|\sigma_r-\sigma_\varphi|-\sigma_s=\left|\frac{E}{1-\nu}\left(\frac{u}{r}-\alpha T\right)-\right.$$
$$\left.-\frac{1-2\nu}{1-\nu}\sigma_r\right|-\sigma_s\leqslant 0,$$

and the quantity $q(\theta,\ r_1)$ (1.14) as the load

$$q(\theta,r_1)=\frac{2sE_ch^2}{R^2\sqrt{3(1-\nu_c^2)}}, \tag{1.15}$$

which is formally the product of the external critical pressure for a homogeneous isotropic spherical shell by a certain factor $s < 1$. Here $h = r_2 - r_1$ is the shell thickness, $R = 1/2(r_1 + r_2)$ the radius of its missile surface, and E_c and ν_c the elastic moduli of the shell material.

To use expression (1.15) under constraint (1.14) for a multilayered shell the elastic moduli of a packet averaged with respect to the thickness [8]

$$\nu_c=B_2/B_1,\ E_c=(B_1-\nu_cB_2)/h, \tag{1.16}$$

are considered as E_c and ν_c, where $B_1=\int\limits_0^1\frac{E(\theta)}{1-\nu^2(\theta)}(r_2-r_1)\,dx;\ B_2=\int\limits_0^1\frac{\nu(\theta)E(\theta)}{1-\nu^2(\theta)}(r_2-r_1)\,dx.$

Taking account of (1.15) and (1.16), the constraint (1.14) can now be represented in the form

$$F_2[\theta,r_1]=p_2-p_1-\int\limits_0^1 Q(\theta,r_1)\,dx\leqslant 0$$
$$\left(Q(\theta,r_1)=\frac{2sh^2E(\theta)[1-\nu_c\nu(\theta)]}{R^2[1-\nu^2(\theta)]\sqrt{3(1-\nu_c^2)}}\right). \tag{1.17}$$

2. NECESSARY OPTIMALITY CONDITIONS

To derive them in the problem (1.1)-(1.17), an expression must be constructed for variations of the target functional (1.12) and the constraints (1.13) and (1.17) in terms of variations of the control $\{\theta(x),\ r_1\}$. To this end we transform the boundary-value problem (1.1)-(1.6). We first integrate (1.2). We have (the prime denotes the derivatives with respect to the coordinate x)

$$r^2\lambda(x)T'(x)=c(r_2-r_1), \tag{2.1}$$

where the variable r is associated with x by the relationships (1.8). Using the condition (1.6) for continuity of the heat flux over the construction layers, we obtain that the constant of integration c will be identical over the whole segment [0, 1]. Now, making the substitution

$$T(x)=cT_0(x)+T_1, \tag{2.2}$$

we obtain a Cauchy problem to determine the function $T_0(x)$ from the relationships (1.5), (2.1), and (2.2)

$$T_0'(x)=\frac{r_2-r_1}{r^2\lambda(x)},\quad T_0(0)=0. \tag{2.3}$$

As a solution of the Cauchy problem (2.3), the function $T_0(x)$ is here continuous in the segment [0, 1]; therefore, the temperature function $T(x)$ (2.2) is also continuous. The constant c is determined from the second condition (1.5) and the relationships (2.1) and (2.2)

$$c = kr_2^2(T_2 - T_1)/[kr_2^2 T_0(1) + 1]. \tag{2.4}$$

For convenience we denote the value of the function $T_0(1)$ by γ. As follows from (2.3), the value of the parameter γ depends on the selection of the control $\{\theta(x), r_1\}$.

The form of Eqs. (1.1), (1.3), (2.3) and the connection condition (1.6) permit introduction of phase variables continuous in the segment [0, 1]

$$\mathbf{Z}(x) = (u(x),\ \sigma_r(x),\ T_0(x))^{\mathrm{T}}. \tag{2.5}$$

Now the original boundary value problem (1.1), (1.3), (1.4), (2.2)-(2.4) can be represented in the form of a boundary-value problem in the unknowns $\mathbf{Z}(x)$ (2.5)

$$\mathbf{Z}'(x) = A(x,\ \gamma,\ \theta,\ r_1)\cdot\mathbf{Z}(x) + \mathbf{B}(x,\ \theta,\ r_1),\quad z_2(0) = -p_1,\ z_2(1) = -p_2,\ z_3(0) = 0, \tag{2.6}$$

where the nonzero elements a_{ij} and b_i of the matrix $A(x, \gamma, \theta, r_1)$ and the vector $\mathbf{B}(x, \theta, r_1)$ are expressed as

$$a_{11} = \frac{2\nu(r_2 - r_1)}{r(\nu - 1)},\quad a_{12} = \frac{(1+\nu)(1-2\nu)(r_2 - r_1)}{E(1-\nu)},\quad a_{13} = \frac{\alpha c(1+\nu)(r_2 - r_1)}{1-\nu},$$
$$a_{21} = \frac{2E(r_2 - r_1)}{r^2(1-\nu)},\quad a_{22} = \frac{(2-4\nu)(r_2 - r_1)}{r(\nu - 1)},\quad a_{23} = \frac{2\alpha Ec(r_2 - r_1)}{r(\nu - 1)},$$
$$b_1 = \alpha T_1(r_2 - r_1)\frac{1+\nu}{1-\nu},\quad b_2 = \frac{2\alpha ET_1(r_2 - r_1)}{r(\nu - 1)},\quad b_3 = \frac{r_2 - r_1}{r^2\lambda}.$$

We replace the local constraint (1.13) by an equivalent integral constraint

$$F_1[\mathbf{Z}, \gamma, \theta, r_1] = \frac{1}{2}\int_{r_1}^{r_2}\{\varphi(\ldots) + |\varphi(\ldots)|\}r^2dr = \int_0^1 \Phi_1(x, \mathbf{Z}, \gamma, \theta, r_1)\,dx = 0. \tag{2.7}$$

Let us note that the functional (2.7) has a Frechet derivative [9] since the integrand $|\varphi(\ldots)|$, which is the absolute value of the Mises plasticity condition, can vanish in a laminar sphere only in a set of zero measure consisting of a finite number of points.

Now, let the pair $\{\theta(x), r_1\}$ be the optimal control from the allowable set (1.10) and (1.11) minimizing the functional (1.12) and satisfying the constraints (1.17) and (2.7). Let us consider the perturbed control $\{\theta^*(x), r_1 + \delta r_1\}$ [9]

$$\theta^*(x) = \begin{cases} g(x), & x \in D,\ g(x) \in U, \\ \theta(x), & x \notin D, \end{cases}\quad r_1 + \delta r_1 \in [a, b],\ |\delta r_1| < \varepsilon \tag{2.8}$$

($D \subset [0, 1]$ is a set of small measure mes$(D) < \varepsilon$; $\varepsilon > 0$ is a small quantity). Using the standard technique [9], the principal parts of the increments of the functionals (1.12), (1.17), and (2.7) can be obtained [for brevity the arguments of the functions referring to the unperturbed control $\{\theta(x), r_1\}$ are omitted

$$\begin{aligned} \delta F[\ldots] &= \int_D \{\Phi(\theta^*, \ldots) - \Phi(\theta, \ldots)\}\,dx + G\delta r_1, \\ \delta F_1[\ldots] &= \int_D \{M(\theta^*, \ldots) - M(\theta, \ldots)\}\,dx + G_1\delta r_1, \\ \delta F_2[\ldots] &= -\int_D \{Q(\theta^*, r_1) - Q(\theta, r_1)\}\,dx - G_2\delta r_1. \end{aligned} \tag{2.9}$$

Here

$$M(x,\ \mathbf{Z},\ \mathbf{\Psi},\ \gamma,\ \theta,\ r_1) = \Phi_1(x,\ \mathbf{Z},\ \gamma,\ \theta,\ r_1) + \mathbf{\Psi}^{\mathrm{T}}(x)\cdot[A(x,\ \gamma,\ \theta,\ r_1)\cdot\mathbf{Z}(x) + \mathbf{B}(x,\ \theta,\ r_1)];$$

$$G = \int_0^1 \frac{\partial}{\partial r_1}\Phi(x, \theta, r_1)\,dx,\quad G_1 = \int_0^1 \frac{\partial}{\partial r_1}M(x, \mathbf{Z}, \mathbf{\Psi}, \gamma, \theta, r_1)\,dx$$

$$G_2 = \int_0 \frac{\partial}{\partial r_1} Q(\theta, r_1)\, dx;$$

and the vector of the conjugate variables $\Psi(x)$ satisfies the boundary-value problem

$$\Psi'(x) = -A^T(x, \gamma, \theta, r_1)\cdot\Psi(x) - \left[\frac{\partial}{\partial \mathbf{Z}}\Phi_1(x, \mathbf{Z}, \gamma, \theta, r_1)\right]^T,$$
$$\psi_1(0) = \psi_1(1) = 0, \quad \psi_3(1) = \int_0^1 \frac{\partial}{\partial\gamma} M(x, \mathbf{Z}, \Psi, \gamma, \theta, r_1)\, dx. \tag{2.10}$$

Let us now compile the expanded functional

$$J[\theta, r_1] = F[\theta, r_1] + \lambda_4 F_1[\mathbf{Z}, \gamma, \theta, r_1] + \lambda_1\{F_2[\theta, r_1] + \xi_1^2\} + \lambda_2\{a - r_1 + \xi_2^2\} + \lambda_3\{r_1 - b + \xi_3^2\} \tag{2.11}$$

(λ_i, ξ_i^2 are Lagrange multipliers and penalty variables [10]). The variation of the functional $J[\theta, r_1]$ (2.11) can be represented by using (2.9) in the form

$$\delta J[\ldots] = \int_D \{H(\theta, \ldots) - H(\theta^*, \ldots)\}\, dx + \{G + \lambda_4 G_1 - \lambda_1 G_2 + \lambda_3 - \lambda_2\}\delta r_1 + 2\sum_{i=1}^{3}\lambda_i\xi_i\delta\xi_i, \tag{2.12}$$

where

$$H(x, \mathbf{Z}, \Psi, \gamma, \theta, r_1) = -\Phi(x, \theta, r_1) - \lambda_4 M(x, \mathbf{Z}, \Psi, \gamma, \theta, r_1) + \lambda_1 Q(\theta, r_1). \tag{2.13}$$

Since the control $\{\theta(x), r_1\}$ is optimal (minimizing), for any allowable controls $\{\theta^*(x), r_1 + \delta r_1\}$ the condition $\delta J[\ldots] \geq 0$ should be satisfied. Then, by virtue of the arbitrariness of the variations δr_1 and $\delta\xi_i$, we obtain from (2.12) the relationships [10]

$$G + \lambda_4 G_1 - \lambda_1 G_2 - \lambda_2 + \lambda_3 = 0; \tag{2.14}$$

$$\lambda_1 F_2[\theta, r_1] = 0, \ \lambda_1 \geqslant 0; \tag{2.15}$$

$$\lambda_2(a - r_1) = 0, \ \lambda_3(r_1 - b) = 0, \ \lambda_2 \geqslant 0, \ \lambda_3 \geqslant 0 \tag{2.16}$$

and because the set of small measure D can be compactly arranged in the segment [0, 1] almost everywhere, the maximum condition for the Hamiltonian function H(...) (2.13) in the argument θ [9] should be satisfied for almost all $x \in [0, 1]$

$$H(x, \mathbf{Z}, \Psi, \gamma, \theta, r_1) = \max_{\theta^* \in U} H(x, \mathbf{Z}, \Psi, \gamma, \theta^*, r_1). \tag{2.17}$$

Therefore, we obtain that the optimal control $\{\theta(x), r_1\}$ and its corresponding optimal trajectory $\Psi(x)$ and the conjugate variable vector $\mathbf{Z}(x)$ should satisfy the boundary-value problems (2.6), (2.10), the relationships and constraints (1.9)-(1.11), (1.17), (2.7), (2.15), (2.16) and the optimality conditions (2.14) and (2.17).

3. COMPUTATIONAL ALGORITHM

The main idea of the direct method of solving the optimal design problem is the construction of a sequence of controls $\{\theta(x), r_1\}_j$, j = 1, 2, ..., that minimizes the target functional (1.12). To do this by introducing a uniform mesh $\{x_i\}$ we partition the segment [0, 1] into n segments D_i simulating a set of small measure. We give the initial control $\{\theta(x), r_1\}$ from the allowable domain (1.9)-(1.11). The function $\theta(x)$ is evidently piecewise-constant with the constancy sections $D_i = [x_i, x_{i+1})$ on which it takes on values from the set U (1.10). The next approximation $\{\theta^*(x), r_1 + \delta r_1\}$ is given in a certain set D_i in the form (2.8)

$$\theta^*(x) = \begin{cases}\theta_k, & x \in D_i, \quad \theta_k \in U,\\ \theta(x), & x \notin D_i;\end{cases} \tag{3.1}$$

$$r_1 + \delta r_1 \in [a, b], \ |\delta r_1| < \varepsilon \tag{3.2}$$

and is determined from the linearized optimization problem: Find that allowable perturbation $\{\theta_k, \delta r_1\}$ in the set D_i that will assure a maximal decrease in the functional F[...] (1.12) (or, equivalently, the minimum of the variation δF[...] (2.9)) under conditions (3.1) and (3.2) and the linearized constraints (1.17) and (2.7)

$$F_1[\mathbf{Z}+\delta\mathbf{Z},\ \gamma+\delta\gamma,\ \theta^*,\ r_1+\delta r_1]\approx F_1[\mathbf{Z},\ \gamma,\ \theta,\ r_1]+ \\ +\delta F_1[\mathbf{Z},\ \gamma,\ \theta,\ r_1]=0; \tag{3.3}$$

$$F_2[\theta^*,\ r_1+\delta r_1]\approx F_2[\theta,\ r_1]+\delta F_2[\theta,\ r_1]\leqslant 0, \tag{3.4}$$

where the expressions for $\delta F_1[\ldots]$ and $\delta F_2[\ldots]$ are given by (2.9). This linearized problem is a modification of the problem examined in Secs. 1 and 2. We hence obtain directly that the optimal perturbation $\{\theta_k, \delta r_1\}$ should satisfy the relationships

$$\delta r_1=-\tau\{G+\lambda_4 G_1-\lambda_1 G_2-\lambda_2+\lambda_3\},\ \tau\geqslant 0; \tag{3.5}$$

$$\lambda_1\{F_2[\theta,\ r_1]+\delta F_2[\theta,\ r_1]\}=0,\ \lambda_1\geqslant 0; \tag{3.6}$$

$$\lambda_2(a-r_1-\delta r_1)=0,\ \lambda_3(r_1+\delta r_1-b)=0,\ \lambda_2\geqslant 0,\ \lambda_3\geqslant 0 \tag{3.7}$$

and the constraints (3.2)-(3.4).

The multipliers τ, λ_2, and λ_3 are found from (3.2) and (3.7) during the numerical computation process. One of two modifications is realized for the determination of the best correction θ_k (3.1).

A. $G_1 \neq 0$. We then obtain from (3.3) and (3.5)

$$\delta r_1=-\Big\{\int\limits_{D_i}[M(\theta_k,\ \ldots)-M(\theta,\ \ldots)]\,dx+F_1[\mathbf{Z},\ \gamma,\ \theta,\ r_1]\Big\}/G_1,$$

and the correction θ_k minimizing the variation δF[...] (2.9) is found from the condition

$$\int\limits_{D_i}H(x,\ \mathbf{Z},\ \mathbf{\Psi},\ \gamma,\ \theta_k,\ r_1)\,dx=\max_{\theta_j\in U}\int\limits_{D_i}H(x,\ \mathbf{Z},\ \mathbf{\Psi},\ \gamma,\ \theta_j,\ r_1)\,dx$$
$$(H(x,\ \mathbf{Z},\ \mathbf{\Psi},\ \gamma,\ \theta_j,\ r_1)=-\Phi(x,\ \theta_j,\ r_1)+GM(x,\ \mathbf{Z},\ \mathbf{\Psi},\ \gamma,\ \theta_j,\ r_1)/G_1).$$

Here (3.4) should be satisfied as well as the constraints (3.2) and (3.7).

B. $G_1 = 0$. Assuming the exact equality (3.4) is satisfied, we find from relationships (3.4) and (3.5) ($G_2 \neq 0$ in the problem under consideration)

$$\delta r_1=-\Big\{\int\limits_{D_i}[Q(\theta_k,\ r_1)-Q(\theta,\ r_1)]\,dx-F_2[\theta,\ r_1]\Big\}\Big/G_2.$$

Here the multiplier is

$$\lambda_1=\Big\{F_2[\theta,\ r_1]-\int\limits_{D_i}[Q(\theta_k,\ r_1)-Q(\theta,\ r_1)]\,dx\Big\}\Big/(\tau G_2^2)+(G-\lambda_2+\lambda_3)/G_2.$$

According to condition (3.6), $\lambda_1 \geq 0$. If $\lambda_1 \geq 0$, then the correction θ_k is determined from the expression

$$\int\limits_{D_i}H(x,\ \theta_k,\ r_1)\,dx=\max_{\theta_j\in U}\int\limits_{D_i}H(x,\ \theta_j,\ r_1)\,dx$$
$$(H(x,\ \theta_j,\ r_1)=-\Phi(x,\ \theta_j,\ r_1)+GQ(\theta_j,\ r_1)/G_2)$$

with the constraints (3.2), (3.7) and $F_1[\mathbf{Z} + \delta\mathbf{Z},\ \gamma + \delta\gamma,\ \theta^*,\ r_1 + \delta r_1] = 0$ (3.3) taken into account. If $\lambda_1 < 0$, then the assumption about compliance with the exact equality (3.4) is not true. Then the constraint (3.4) is not taken into account and the correction $\{\theta_k, \delta r_1\}$ is determined from the relations

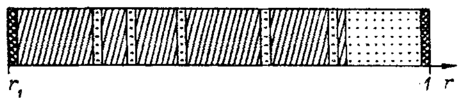

Fig. 1

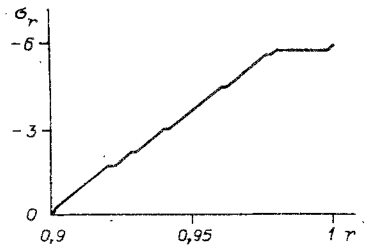

Fig. 2

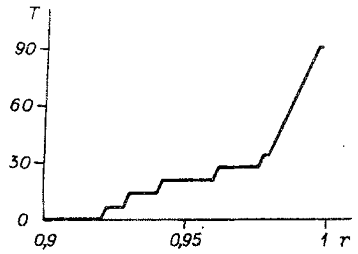

Fig. 3

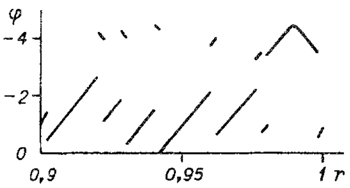

Fig. 4

$$\delta r_1 = -\tau[G - \lambda_2 + \lambda_3], \quad \int\limits_{D_i} \Phi(x, \theta_k, r_1)\, dx = \min_{\theta_j \in U} \int\limits_{D_i} \Phi(x, \theta_j, r_1)\, dx$$

with the constraints (3.2), (3.3), and (3.7) taken into account.

By constructing the new control $\{\theta^*(x),\ r_1 + \delta r_1\}$ in this manner we take it as the initial control and construct the next approximation. The process is considered terminated and this mesh of partitions $\{x_i\}$ if the control $\{\theta(x),\ r_1\}$ does not change in any of the sets D_i.

4. EXAMPLES

The set W consists of five materials with the following mechanical and physical dimensionless characteristics (1.7): E = 270, 7100, 10,500, 21,000, and 11,200; ν = 0.27, 0.3, 0.3, 0.3, and 0.33; ρ = 0.65, 2.85, 4.4, 7.8, and 8.93; σ_s = 4.5, 40, 60, 120, and 20; $\alpha \cdot 10^6$ = 100, 21.94, 8.4, 15, and 16.7; λ = 0.07, 155.4, 8.4, 45.4, and 389.6; k = 23.26. Given on the inner surface of the sphere whose radius r_1 can vary between the limits of the segment [0.7, 0.91] are $p_1 = 0$ and $T_1 = 0$. On the outer sphere surface whose radius r_2 is considered fixed, equal to one, $p_2 = 6$, and the heat transfer according to a Newton law with the temperature of the external medium $T_2 = 100$ are given. The coefficient is s = 0.1 in (1.15). The sphere is covered inside and out by thin (0.002 thickness) nonvariable shielding layers of a third material. The inner variable domain of the sphere is partitioned into 48 equal parts simulating the set D_i along the thickness.

Taken as the initial approximation is a four-layer sphere with $r_1 = 0.9$, F = 0.2203, and with layers [0.9, 0.912] of the third material, [0.912, 0.972] of the second material, [0.972, 0.998] of the first material, and [0.998, 1] of the third material. Obtained as a result of optimization is a fourteen-layered sphere (included among the layers are also the two nonvariable protective layers) with $r_1 = 0.9015$ and $F_* = 0.202$ [the constraint $F_2 = -0.9$ (1.17)]. A slit of this sphere along the thickness is represented in Fig. 1. Layers from the third material are shaded by hatching, from the second material by lines, and from the first material by dots. Graphs of the distribution of the radial stresses $\sigma_r(r)$, the temperature T(r), and the stress intensity function $\varphi(r)$ (1.13) are presented in Figs. 2-4. A three-layered sphere with $r_1 = 0.91$ and $F^* = 0.2398$ whose inner variable domain consists of the second material is the lightest "homogeneous" sphere satisfying the constraints on strength (1.13) and stability (1.14) for given p_1, p_2 and T_1, T_2. The relative gain in weight for an optimal sphere as compared with the given "homogeneous" sphere is $(1 - F_*/F^*) \cdot 100\% = 15.8\%$.

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A CLASS OF INVERSE CREEP THEORY PROBLEMS

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Certain inverse problems associated with finding the external effects needed to obtain the requisite residual body or plate shape under creep conditions in a given time t_* with elastic unloading are taken into account at the time $t = t_*$. It is assumed here that the unknown external effects belong to a definite class, for instance relaxation problems were examined in [2, 4, 5] when unknown displacements of body surface points (unknown plate deflections) remained fixed during the time t_*, and external loads were considered constant in time in [1, 2], etc.

A class of inverse problems about finding external loads such as would assure a given residual body (plate) shape at any running time is investigated in this paper. A theorem on the uniqueness of the solution is proved for the cases of small strains. A variational formulation is given for these problems on the basis of finding the stationary value of a certain functional; the displacement and stress velocities are here varied simultaneously as both running and residual (after elastic unloading). The solution of the problem in an exact formulation is compared in a specific example with the solution obtained by using the mentioned mixed variational principle.

1. Let us consider a uniformly heated body of volume v with surface S whose governing strain equations we write as

$$\varepsilon_{kl} = a_{klmn}\sigma_{mn} + \varepsilon^c_{kl} \qquad (k, l = 1, 2, 3), \tag{1.1}$$

where $\varepsilon_{k\ell}$, $\varepsilon_{k\ell}{}^c$, $\sigma_{k\ell}$, $a_{k\ell mn} = a_{mnk\ell}$ are components of the total strain, creep strain, stress and elastic pliability tensors, respectively, summation from 1-3 is performed over repeated

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